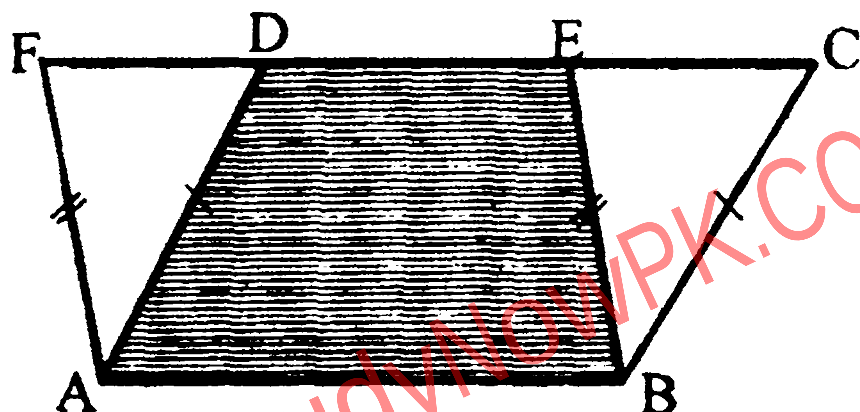
Unit 16
Theorems Related With Area

THEOREM 16.1.1

Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area.

Solution:



Given:

Two parallelograms ABCD and ABEF having the same base AB and between the same parallel lines AB and DE.

To Prove:

Area of parallelogram ABCD = Area of parallelogram ABEF

Proof:

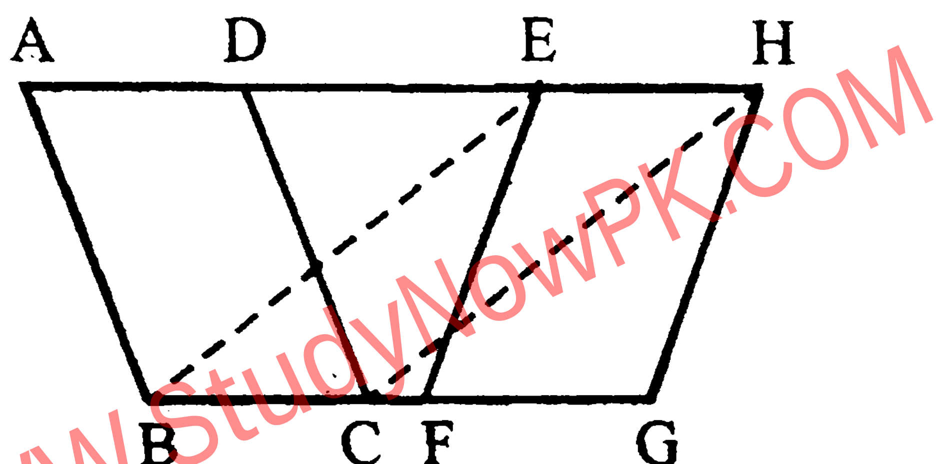
Statements	Reasons
area of (parallelogram ABCD) = area of (quadrilateral ABED) + area of (ΔCBE)... (1)	Area addition axiom
= area of (quadrilateral ABED) + area of (ADAF) .... (2)	Area addition axiom
$m\overline{CB} = m\overline{DA}$	Opposite sides of a parallelogram
$m\overline{BE} = m\overline{AF}$	Opposite sides of a parallelogram
[illegible]	Opposite sides of a parallelogram

∴ $\Delta CBE \cong \Delta DAF$	S.A.S. congruent Axiom
area of (ΔCBE) = area of (ΔDAF) .... (3)	Congruent area axiom
Hence area of (parallelogram ABCD) = area of (parallelogram ABEF)	from (1), (2) and (3)

THEOREM 16.1.2

Parallelograms on equal bases and having the same (or equal) altitude are equal in area.

Solution:



Given:

Parallelograms ABCD, EFGH are on the equal bases BC, FG, having equal altitudes.

To Prove:

area of (parallelogram ABCD) = area of (parallelogram EFGH)

Construction:

Place the parallelograms ABCD and EFGH so that their equal bases $\overline{BC}$, $\overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$

Proof:

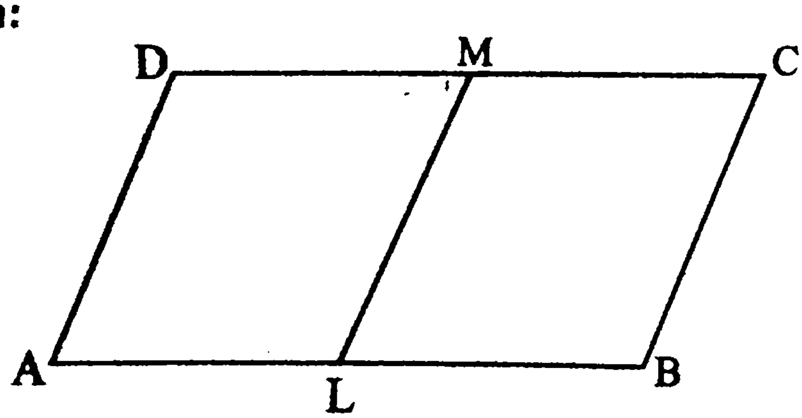
Statements	Reasons
The given ||gms ABCD and EFGH are between the same parallels	Their altitudes are equal (given)

Hence ADEH is a straight line ∥ to BC	
$\therefore \quad m\overline{BC} = m\overline{FG}$ $= m\overline{EH}$	Given EFGH is a parallelogram
Now $m\overline{BC} = m\overline{EH}$ and they are ∥ $\therefore$ BE and CH are both equal and ∥	
Hence EBCH is a parallelogram	A quadrilateral with two opposite
Now $\|^{gm}$ ABCD = $\|^{gm}$ EBCH (i)	Being on the same base BC and between the same parallels
But $\|^{gm}$ EBCH = $\|^{gm}$ EFGH (ii)	Being on the same base EH and between the same parallels
Hence area ($\|^{gm}$ ABCD) = area ($\|^{gm}$ EFGH)	From (i) and (ii)

EXERCISE 16.1

Q1. Show that the line segment joining the mid-points of opposite sides of a parallelogram, divides it into two equal parallelograms.

Solution:



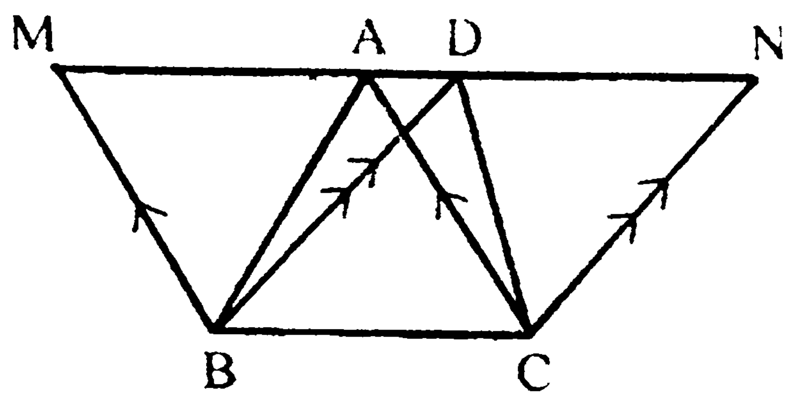
Given:

ABCD is a parallelogram. L is mid point of $\overline{AB}$ and M is mid point of $\overline{DC}$.

THEOREM 16.1.3

Triangles on the same base and of the same (i.e., equal) altitudes are equal in area.

Solution:



Given:

Δs ABC, DBC on the same base $\overline{BC}$, and having equal altitudes.

To Prove:

area of (ΔABC) = area of (ΔDBC)

Construction:

Draw $\overline{BM}$ II to $\overline{CA}$, $\overline{CN}$ II to $\overline{BD}$ meeting $\overleftrightarrow{AD}$ produced in M, N.

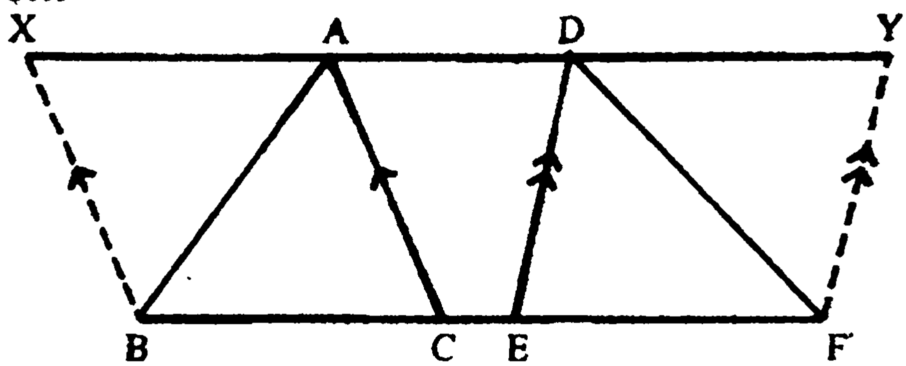
Proof:

Statements	Reasons
ΔABC, Δ DEF are between the same $\|^{gm}$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$ ∴ area ($\|^{gm}$ BCAM) = area ($\|^{gm}$ BCND) ....(i)	These $\|^{gms}$ are on same base $\overline{BC}$ and between the same $\|^{s}$
But ΔABC = $\frac{1}{2}$ ($\|^{gm}$ BCAM) (ii)	Each diagonal of a $\|^{gm}$ bisects it into two congruent triangles
and ΔDEF = ($\|^{gm}$EFYD) (iii)	
Hence Area (Δ ABC) = Area (Δ DBC)	From (i), (ii) and (iii)

THEOREM 16.1.4

Triangles on equal bases and of equal altitudes are equal in area.

Solution:



Given:

Δs ABC, DEF on equal bases BC, EF and having altitudes equal.

To prove:

Area (Δ ABC) = Area (Δ DEF)

Construction:

Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it. Draw BX II to CA and FY II to ED meeting AD produced in X, Y respectively.

Proof:

Statements	Reasons
Δ ABC, Δ DEF are between the same parallels	Their altitudes are equal (given)
$\therefore$ XADY is II to BCEF	
$\therefore$ area (II^{gm} BCAX) = area (II^{gm} EFYD) ......(i)	These II^{gms} are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2}$ (II^{gm} BCAX) (ii)	Diagonal of a II^{gm} bisects it
and $\Delta DEF = \frac{1}{2}$ (II^{gm} EFYD) (iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i), (ii) and (iii)

Hence ADEH is a straight line ॥ to BC	
$\therefore$ $m\overline{BC} = m\overline{FG}$	Given
$= m\overline{EH}$	EFGH is a parallelogram
Now $m\overline{BC} = m\overline{EH}$ and they are ॥	
$\therefore$ BE and CH are both equal and ॥	
Hence EBCH is a parallelogram	A quadrilateral with two opposite
Now ॥^{gm} ABCD = ॥^{gm} EBCH (i)	Being on the same base BC and between the same parallels
But ॥^{gm} EBCH = ॥^{gm} EFGH (ii)	Being on the same base EH and between the same parallels
Hence area (॥^{gm} ABCD) = area (॥^{gm} EFGH)	From (i) and (ii)

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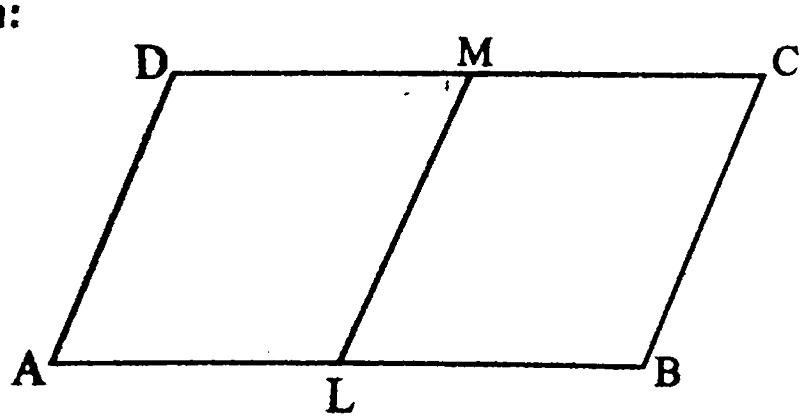


Given:

ABCD is a parallelogram. L is mid point of $\overline{AB}$ and M is mid point of $\overline{DC}$.

To prove:

Area of parallelogram ALMD = Area of parallelogram LBCM.

Proof:

$\overline{AB} \parallel \overline{CD}$ opposite sides of parallelogram ABCD.

As L is mid point of $\overline{AB}$

$\overline{AL} \cong \overline{LB}$

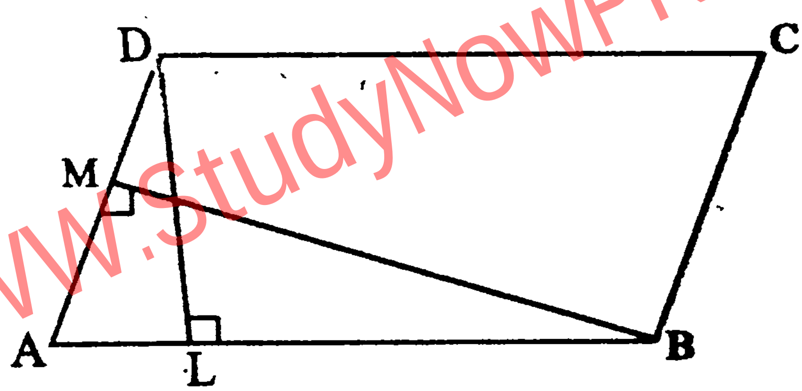
The parallelogram ALMD and LBCD are on equal bases $(\overline{AL} = \overline{LB})$ and between the same parallel lines $\overline{AB}$ and $\overline{DC}$.

∴ They are equal areas

Hence Area of parallelogram ALMD = Area of parallelogram LBCM.

Q2. In a parallelogram ABCD, $m\overline{AB} = 10$ cm. The altitudes corresponding to sides AB and AD are respectively 7 cm and 8 cm. Find $\overline{AD}$.

Solution:



Given:

ABCD is a parallelogram.

$m\overline{AB} = 10$ cm, $\overline{DL}$ and $\overline{BM}$ are altitudes

$m\overline{DL} = 7$ cm, $m\overline{BM} = 8$ cm

To prove:

$m\overline{AD} = ?$

Proof:

Area of a parallelogram = base × altitude

Area of a parallelogram ABCD

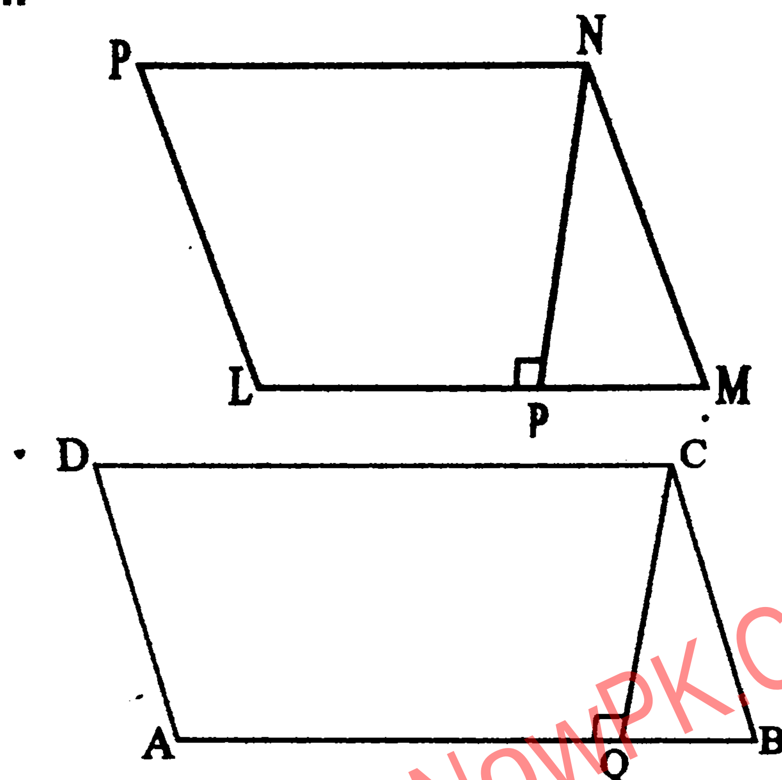
$m\overline{AB} \times m\overline{DL} = m\overline{AD} \times m\overline{BM}$

$10 \times 7 = m\overline{AD} \times 8$

$m\overline{AD} = \frac{10\times7}{8} = \frac{35}{8} = 8.75$ cm

Q3. If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.

Solution:



Given:

In a parallelogram ABCD, $\overline{CQ}$ is altitude and in parallelogram LMNP, $\overline{NP}$ is altitude. Areas of parallelogram ABCD = Area of parallelogram LMNP and $m\overline{AB} = m\overline{LM}$

To prove:

$m\overline{CQ} = m\overline{NP}$.

Proof:

Area of a parallelogram ABCD = Area of parallelogram LMNP (Given)

We know that area of a parallelogram= base ×altitude

$m\overline{AB} \times m\overline{CQ} = m\overline{LM} \times m\overline{NP}$

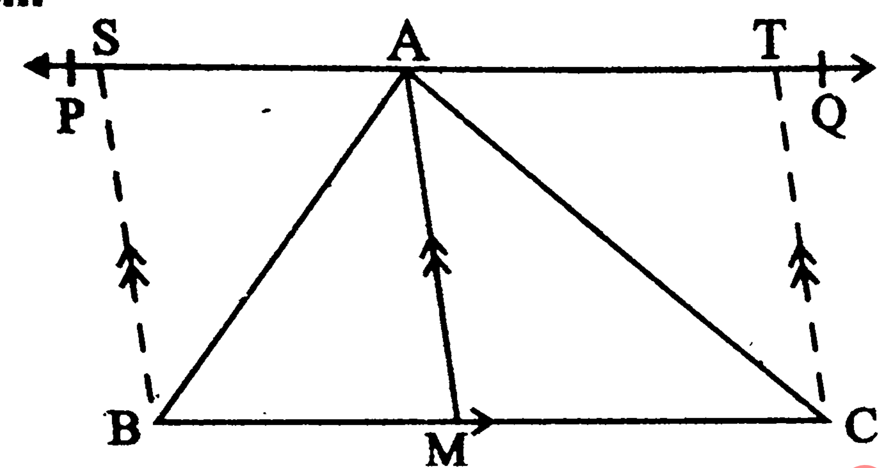
but $m\overline{AB} = m\overline{LM}$ (Given)

$m\overline{CQ} = m\overline{NP}$

EXERCISE 16.2

Q1. Show that a median of a triangle divides it into two triangles of equal area.

Solution:



Given:

In ΔABC, $\overline{AM}$ is median

i.e. $m\overline{BM} = m\overline{MC}$

To prove:

Area ΔABM = area ΔACM

Construction:

Draw $\overline{PQ} \parallel \overline{BC}$, Draw $\overline{BS} \parallel \overline{AM}$ and $\overline{CT} \parallel \overline{AM}$

Proof:

$\overline{BS} \parallel \overline{MA}$ (Construction)

$\overline{BM} \parallel \overline{SA}$ (Construction)

$\therefore$ BMAS is a parallelogram.

Similarly AMCT is a parallelogram.

Parallelograms BMAS and they are between the same parallel lines $\overline{BC}$ and $\overline{PQ}$.

$\therefore$ They have equal areas.

So Area parallelogram BMAS=Area parallelogram AMCT

$\Rightarrow$ $\frac{1}{2}$(area parallelogram BMAS)

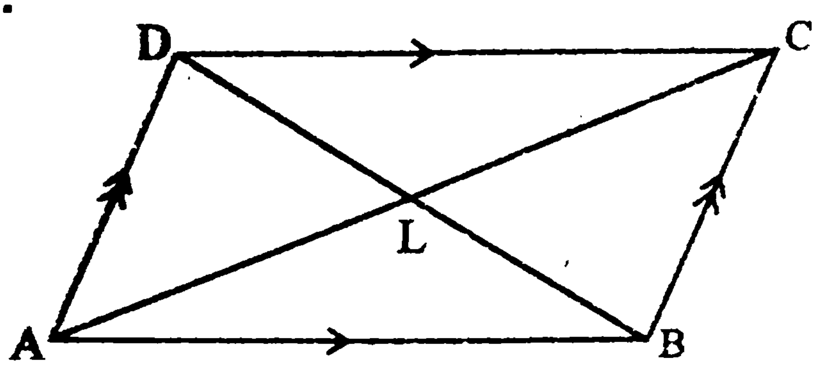
$= \frac{1}{2}$(area parallelogram AMCT)

$\Rightarrow$ Area ΔABM = Area ΔAMC

So a median of a triangle divides it into two triangles of equal area.

Q2. Prove that a parallelogram is divided by its diagonals into four triangles of equal area.

Solution:



Given:

In parallelogram $ABCD$, $\overline{AC}$ and $\overline{BD}$ are its diagonal, which meet at L.

To prove:

Triangles ABL, BCL, CDL and ADL have equal area.

Proof:

Triangles ABC and ABD have the same base $\overline{AB}$ and are between the same parallel lines $\overline{AB}$ and $\overline{DC}$.

They have equal area.

or Area ΔABC = Area ΔABD

or Area ΔABL + Area ΔBCL = Area ΔABL + Area ΔADL

$\Rightarrow$ Area ΔBCL = Area ΔADL (i)

Similarly Area ΔABC = Area ΔBCD

$\Rightarrow$ Area ΔBCL = Area ΔABL

Area ΔBCL = Area ΔCDL

$\Rightarrow$ Area ΔABL = Area ΔCDL (ii)

As diagonals of a parallelogram bisect each other.

L is mid point of $\overline{AC}$.

So $\overline{BL}$ is a median of ΔABC

Area ΔABL = Area ΔBCL (iii)

From (i), (ii) and (iii) we get

Area ΔABL = Area ΔBCL = Area ΔCDL = Area ΔADL

Q3. Divide a triangle into six equal triangular parts.

Solution:

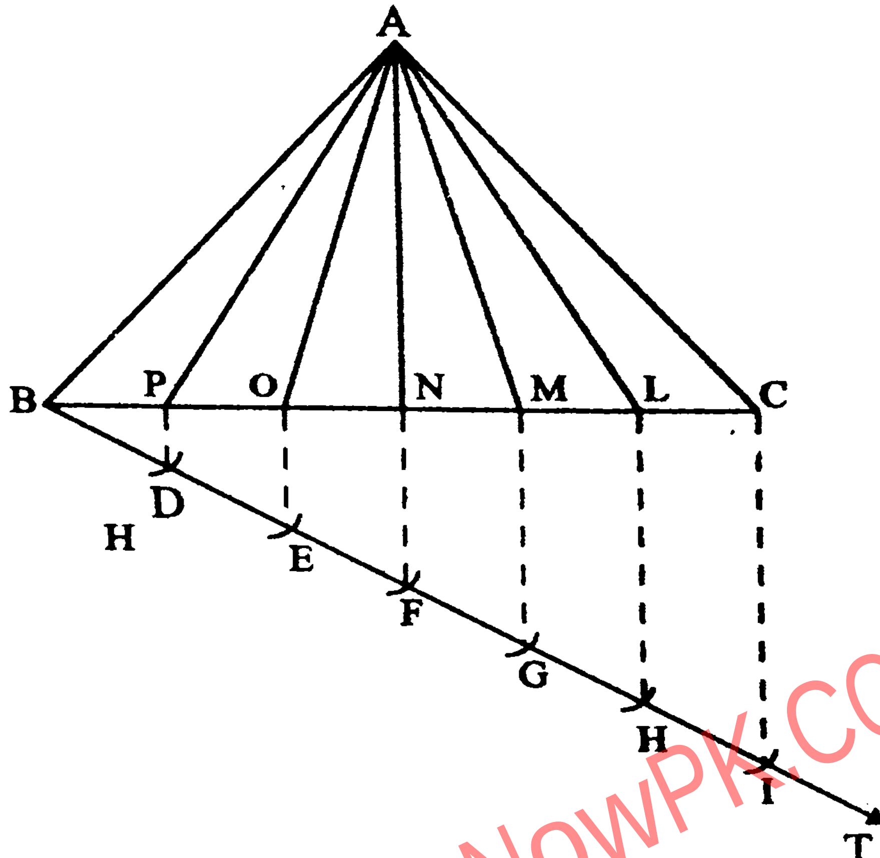
Given:

Δ ABCD

Required:

To divide ΔABC into sic equal triangular parts.

Construction:



(i) Draw the ray $\overrightarrow{BT}$ making an acute angle CBT.

(ii) On $\overline{BT}$ mark six points D; E; F; G; H and I such that $m\overline{BD} = m\overline{DE} = m\overline{EF} = m\overline{FG} = m\overline{GH} = m\overline{HI}$

(iii) Join IC.

(iv) Draw $\overline{HL}, \overline{GM}, \overline{FN}, \overline{EO}, \overline{DP}$ each parallel to $\overline{IC}$.

(v) Join A to L, M, N, O and P. So BAP, PAO, OAN, NAM, MAL and LAC are required six equal parts.

REVIEW EXERCISE 16

Q1. Which of the following are true and which are false?

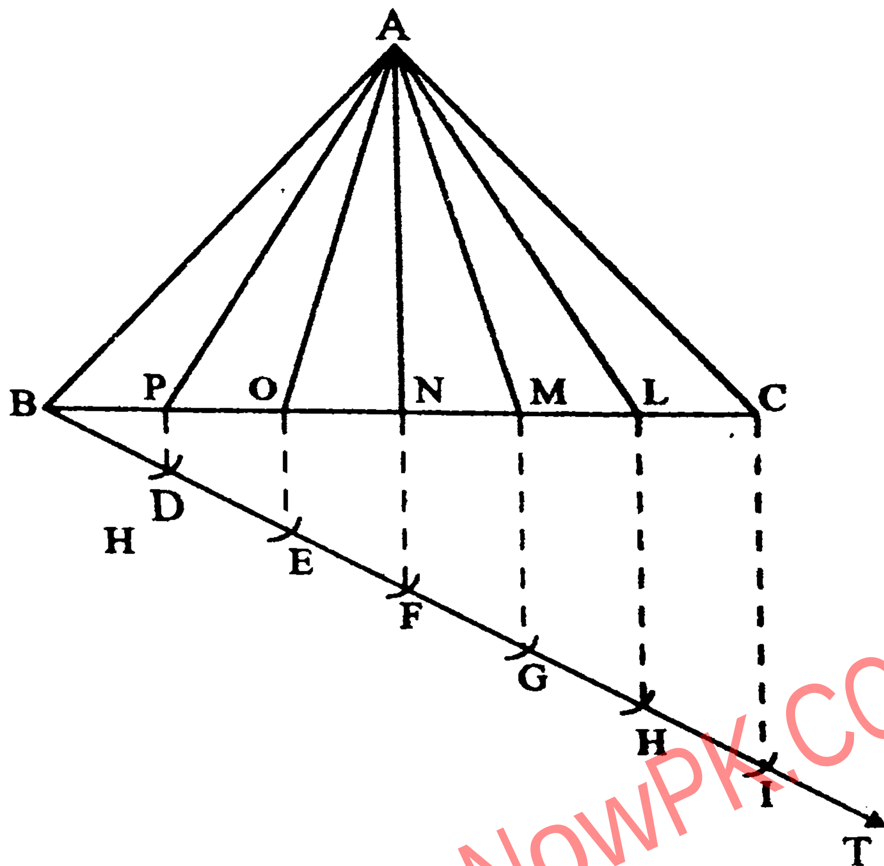
(i) Area of a figure means region enclosed by bounding lines of closed figure.

(ii) Similar figure have same area.

(iii) Congruent figures have same area.

(iv) A diagonal of a parallelogram divides it into two non-congruent triangles.

Construction:



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(i) Area of a figure means region enclosed by bounding lines of closed figure.

(ii) Similar figure have same area.

(iii) Congruent figures have same area.

(iv) A diagonal of a parallelogram divides it into two non-congruent triangles.

(v) Altitude of a triangle means perpendicular from vertex to the opposite side (base).

(vi) Area of parallelogram is equal to the product of base and height.

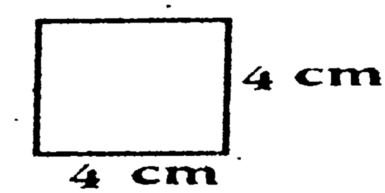
Answers:

(i) T	(ii) F	(iii) T	(iv) F	(v) T	(vi) T

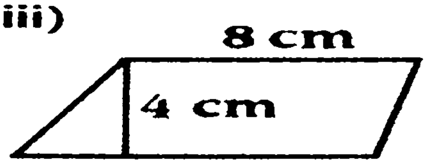
Q2. Find the area of the following.

(i)

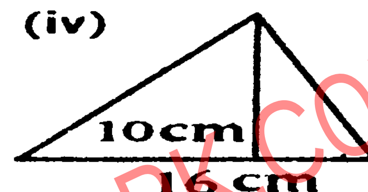
(ii)



(iii)



(iv)



Solution:

(i) Area $= 6 \times 3 = \mathbf{18\ cm^2}$

(ii) Area $= 4 \times 4 = \mathbf{16\ cm^2}$

(iii) Area $= 8 \times 4 = \mathbf{32\ cm^2}$

(iv) Area $= \frac{1}{2} \times 10 \times 16 = \mathbf{80\ cm^2}$

Q3. Define the following.

Solution:

(i) Area of a figure:

The region enclosed by the bounding lines of a closed figure is called the area of the figure.

The area of a closed region is expressed in square units (say, sq. m or m^2)

(ii) Triangular Region:

The interior of a triangle is the part of the plane enclosed by the triangle.

A triangle region is the union of a triangle and its interior i.e., the three line segment forming the triangle and its interior.

By area of a triangle, we mean the area of its triangular region.

(iii) Rectangular Region:

The interior of a rectangle is the part of the plane enclosed by the rectangle.

A rectangular region is the union of a rectangle and it's interior.

A rectangular region can be divided into two or more than two triangular regions in many ways.

(iv) Altitude or Height of a triangle

If one side of a triangle is taken as its base the perpendicular to that side, from the opposite vertex is called altitude or height of the triangle.